DETERMINATION OF THE TEMPERATURE FIELD OF A WEDGE DURING HEATING AND COOLING

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The problem of nonlinear heat conduction is solved numerically by the Piesman–Rackford method for a wedge heated with moving sources of various intensities or cooled.

The basis for calculating the temperature field of a wedge has been established in several studies dealing with the subject [1-3]. The method shown there does not cover accurately enough the case where the heat sources are distributed over the volume and the thermophysical properties are nonlinear.

An application of the numerical method makes it possible to determine the temperature field of a wedge of any shape with a volume distribution of heat sources moving at various velocities, and to take into account the appreciable effect of nonlinearity under transient conditions in thermal processes.

The heating of a wedge occurs in technical situations most often when it is placed in a stream with a continuous distribution of heat sources of various intensities.

Cooling of a wedge begins when heat is transferred to the ambient medium not containing heat sources and having a temperature much below the wedge temperature.

The equation of heat conduction written in terms of finite differences is solved in this case by the absolutely-stable Piesman–Rackford method, which makes it possible to remove any additional constraints on the correspondence between steps in the space grid covering the region under study and steps along the time coordinate. When applying the Piesman–Rackford method, one can write the equation of heat conduction in terms of finite differences along the rows

$$\frac{\Theta_{ij}^{k+\frac{1}{2}}-\Theta_{ij}^{k}}{\Delta t}=\frac{\lambda_{i+\frac{1}{2}j}^{k}}{h}\cdot\frac{\Theta_{i+1j}^{k+\frac{1}{2}}-\Theta_{ij}^{k+\frac{1}{2}}}{h}-\frac{\lambda_{i-\frac{1}{2}j}^{k}}{h}\,\frac{\Theta_{ij}^{k+\frac{1}{2}}-\Theta_{i-1j}^{k+\frac{1}{2}}}{h}+v_{x_{ij}}\frac{\Theta_{i+1j}^{k+\frac{1}{2}}-\Theta_{ij}^{k+\frac{1}{2}}}{h}+\frac{\lambda_{ij+\frac{1}{2}}^{k}}{h}$$
$$\times\frac{\Theta_{ij+1}^{k}-\Theta_{ij}^{k}}{h}-\frac{\lambda_{ij-\frac{1}{2}}^{k}}{h}\cdot\frac{\Theta_{ij}^{k}-\Theta_{ij-1}^{k}}{h}+v_{y_{ij}}\frac{\Theta_{ij+1}^{k}-\Theta_{ij}^{k}}{h}+\frac{\Theta_{ij}^{k}}{\Delta t},\qquad(1)$$

and along the columns

$$\frac{\Theta_{ij}^{k+1}-\Theta_{ij}^{k+\frac{1}{2}}}{\Delta t}=\frac{\lambda_{i+\frac{1}{2}j}^{k+\frac{1}{2}}}{h}\cdot\frac{\Theta_{i+1j}^{k+\frac{1}{2}}-\Theta_{ij}^{k+\frac{1}{2}}}{h}-\frac{\lambda_{i-\frac{1}{2}j}^{k+\frac{1}{2}}}{h}\cdot\frac{\Theta_{ij}^{k+\frac{1}{2}}-\Theta_{i-1j}^{k+\frac{1}{2}}}{h}+v_{x_{ij}}\frac{\Theta_{i+1j}^{k+\frac{1}{2}}-\Theta_{ij}^{k+\frac{1}{2}}}{h}$$
$$+\frac{\lambda_{ij+\frac{1}{2}}^{k+\frac{1}{2}}}{h}\cdot\frac{\Theta_{ij+1}^{k+1}-\Theta_{ij}^{k+1}}{h}-\frac{\lambda_{ij-\frac{1}{2}}^{k+\frac{1}{2}}}{h}\cdot\frac{\Theta_{ij}^{k+1}-\Theta_{ij-1}^{k+1}}{h}+v_{y_{ij}}\frac{\Theta_{ij+1}^{k+1}-\Theta_{ij}^{k+1}}{h}+\frac{Q_{ij}^{k+\frac{1}{2}}}{\Delta t}.\qquad(2)$$

Equations (1) and (2) are solved by the elimination method. Since the method used here is of second-order accuracy with respect to time, hence the quasilinear equation of heat conduction can be linearized with a sufficiently small error only if the time step Δt and the grid step h are selected sufficiently small.

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In the beginning both steps are selected as follows:

$$\frac{4\Delta t\lambda}{h^2} \leqslant 1; \tag{3}$$

$$h \leqslant \frac{\lambda}{v_x}; \quad h \leqslant \frac{\lambda}{v_y}, \tag{4}$$

defining the upper limits of h and Δt for solving the problem by the explicit scheme.

Since here the problem is solved by the implicit scheme, hence conditions (3) and (4) must not necessarily be adhered to. Increasing the steps h and Δt will, however, result in a loss of accuracy but will also yield a saving in computation time. For this reason, a series of experimental computations was made with Δt varied through 2-3 orders of magnitude. The results of these calculations and their comparison with test data have shown that, within the range of conditions which vary during metal cutting, Δt may be selected up to 10 times longer than stipulated by condition (3) without incurring appreciable computation errors.

The most difficult engineering problem is to calculate the temperature field of a wedge when the latter heats up while cutting a layer of metal.

Our method of solving the heat-conduction problem makes it possible to calculate according to Eqs. (1) and (2) directly over the entire region under study. Contact heat transfer between the cutting tool and the machined metal is taken into account by solving system (1)-(2) with the following modification of the thermophysical coefficients at the contact along the horizontal wedge surface:

$$\lambda_{ij+\frac{1}{2}} = \lambda^{\text{wedge}}_{ij+\frac{1}{2}}, \quad \lambda_{ij-\frac{1}{2}} = \lambda^{\text{metal}}_{ij-\frac{1}{2}},$$

$$\lambda_{i+\frac{1}{2}j} = \frac{\lambda^{\text{metal}}_{i+\frac{1}{2}j} + \lambda^{\text{wedge}}_{i+\frac{1}{2}j}}{2}, \quad \lambda_{i-\frac{1}{2}j} = \frac{\lambda^{\text{metal}}_{i-\frac{1}{2}j} + \lambda^{\text{wedge}}_{i-\frac{1}{2}j}}{2}.$$

The same applies, analogously, to the lateral wedge surface.

Heat sources moving at various velocities are distributed over the volume of deformed metal surrounding the wedge, and over the wedge surfaces within the zone of contact with the metal. The temperature of internal heat sources within the deformation zone is determined from

$$Q = k_{\text{d}} \frac{\sigma_i \dot{\varepsilon}_i \Delta t}{Jc\rho}.$$

The temperature due to the surface heat sources generated by friction is determined from

$$Q = \frac{\tau v \Delta t}{Jc\rho h}.$$

The quantities σ_i, $\dot{\varepsilon}_i$, τ, v_x, v_y and their distributions are established by well-known experimental–analytical methods [5]. Equations (1) and (2) are solved with boundary conditions specified at the wedge surfaces and at the edge of the region bounded by plane surfaces, also with the appropriate initial conditions. At the surfaces where metal is chipping off, far from the active heat sources, one stipulates boundary conditions of the second kind $\partial\Theta/\partial n = 0$. At interfaces between wedge or deformed metal and the ambient medium one stipulates boundary conditions of the third kind $\lambda(\partial\Theta/\partial n) = \lambda(\Theta - T)$.

The wedge cooling after a cut is calculated according to the same procedure. Boundary conditions of the third kind are stipulated along the wedge surfaces, however, with Eqs. (1) and (2) transformed accordingly and $v_{x_{ij}}$, $v_{y_{ij}}$, and Θ_{ij} assumed equal to zero here. The rate of wedge cooling is determined by the heat-transfer coefficient α, which depends on the properties of the coolant, on the rate of coolant supply, and on the temperature at the given point on the wedge surface.

The problem is solved on a MINSK-22 computer according to a program set up for calculating a region subdivided by a grid into 1000 points and for printing the output data on these points, which makes it possible to immediately evaluate the temperature field of a wedge.

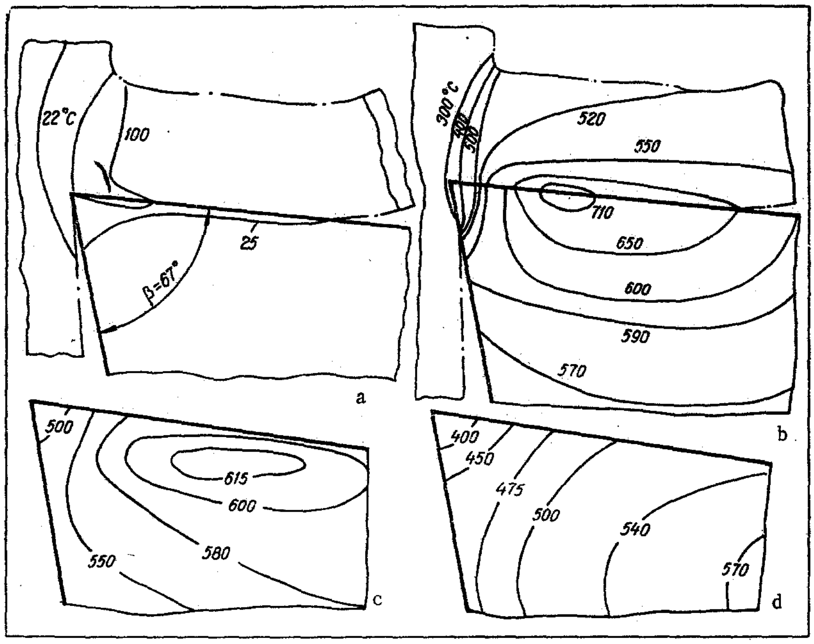

Fig. 1. Variation of the temperature field during heating and cooling of a wedge: a) 0.005 sec after the start of heating; b) 0.05 sec after the start of heating; c) 0.0025 sec after the start of cooling; d) 0.0125 sec after the start of cooling.

As an example, we show here (Fig. 1) the results obtained for a wedge of high-strength alloy with a $\beta = 67°$ nose angle cutting a layer of grade ShKh-15 steel and with aqueous emulsion as the coolant.

Under these conditions the temperature at the wedge surface almost stabilizes within 0.001-0.005 sec after the cutting operation has begun and the temperature field within the contact zone stabilizes within 0.03-0.05 sec. It requires an appreciable amount of computer time to determine the temperature field of an entire wedge after complete stabilization, which depends on the wedge dimensions and on the heat transfer with the ambient medium, but it is often unnecessary as long as only the temperature field of the contact zone is of most interest.

Under the conditions of wedge heating specified here, the temperature of the zone is maximum at the surface with the longest contact and it gradually shifts away from the vertex along the contact line.

The main heat source determining the temperature field of a wedge is friction at the wedge surfaces, while the heat sources inside the metal deformation zone – because of their high velocity – determine the temperature at the vertex only.

If the coolant acts on the wedge surfaces only, then the zone of maximum temperature gradually shifts from the surface deeper into the volume and, at the same time, the absolute level of these temperatures decreases.

Including the nonlinearity due to the temperature-dependence of the thermophysical coefficients improved the accuracy of computations by 5-10% over computations with constant thermophysical coefficients. The relations $c = c(\Theta)$, $\rho = \rho(\Theta)$, $a = a(\Theta)$, $\lambda = \lambda(\Theta)$ were assumed linear.

The results were checked experimentally for the film of momentarily melting pure metals (10^{-8} m thick) on the inner surface of a lengthwise cut wedge, showing a close agreement within 3% with calculated isotherms.

Thus, the described method of calculation yields the temperature field of a wedge of any shape both during heating with moving heat sources of various intensities or during cooling with an ambient medium under a variable heat transfer along the surface.

NOTATION

Θ is the temperature;
T is the ambient temperature;

x, y are the space coordinates;
h is the grid step;
t is the time
Δt is the time step;
n is the normal to wedge surface;
ρ is the density;
c is the specific heat;
a is the thermal diffusivity;
λ is the thermal conductivity;
α is the heat-transfer coefficient;
v_x, v_y are the velocity of heat source along axes X and Y, respectively;
J is the mechanical equivalent of heat;
σ_i is the stress;
$\dot{\varepsilon}_i$ is the strain rate;
τ is the shear stress;
k_d is the coefficient of deformation energy converted into heat;
Q is the temperature of internal sources.

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